

On the resilience of helical magnetic fields to turbulent diffusion and the provocative astrophysical implications

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ABSTRACT

The extent to which large scale magnetic fields are susceptible to decay via turbulent shredding is important for interpreting the need for in situ large scale dynamos in astrophysics and for observationally inferring field strengths compared to kinetic energy. By solving coupled evolution equations for magnetic energy and magnetic helicity in a system initiated with isotropic turbulence and an arbitrarily helical large scale field, we quantify the decay rate of the latter for a bounded or periodic system. We show that energy associated with the non-helical large scale magnetic field decays at least as fast as the kinematically estimated turbulent diffusion rate, but the decay rate of the large scale helical component depends on whether the ratio of its magnetic energy to the turbulent kinetic energy exceeds a critical value given by $M_{1,c} = (k_1/k_2)^2$, where k_1 and k_2 are the wave numbers of the large and forcing scales. Turbulently diffusing helical fields to small scales while conserving magnetic helicity requires a rapid increase in total magnetic energy. As such, only when the helical fields are sub-critical can they so diffuse. When super-critical, the large scale helical field decays slowly, at a rate determined by microphysical dissipation even in the presence of macroscopic turbulence. In effect, the turbulent diffusion of such an initial large scale helical field produces small scale helicity whose amplification abates further turbulent diffusion of that initial field. Two curious implications are that: (1) Standard arguments supporting the need for in situ large scale dynamos based on the otherwise rapid turbulent diffusion of large scale fields require re-thinking since only the non-helical field is so diffused in a closed system. Boundary terms could however provide potential pathways for rapid change of the large scale helical field. (2) Since $M_{1,c} \ll 1$ for $k_1 \ll k_2$, the presence of long-lived ordered large scale helical fields as in extragalactic jets does not guarantee that the magnetic field dominates the kinetic energy.

Key words: magnetic fields; galaxies: jets; stars: magnetic field; dynamo; accretion, accretion disks; cosmology: miscellaneous

1 INTRODUCTION

Many astrophysical sources including galaxies, stars, compact objects, and accretion engines show direct or indirect evidence for large scale magnetic fields (Shukurov 2007). The extent to which these large scale fields survive in the presence of in situ turbulent diffusion and the conditions that determine their diffusion rates constrains the mechanism of their origin. Do the fields result from in situ dynamo generation or could they have been the result of flux freezing from a previous evolutionary phase?

There has been debate over the extent to which 3-D turbulent diffusion of large scale fields is effective and the role that the small scale fields play in its potential suppression. The controversy originated in part from 2-D studies (Cattaneo & Vainshtein 1991) which seemed to suggest suppression. However, magnetic field lines can interchange in 3-D. This distinction

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is implicit in the fact that the formalism for computing the isotropic turbulent diffusion coefficient of large scale fields reveals a suppression in 2-D that is absent in 3-D (Pouquet et al. 1976; Gruzinov & Diamond 1996). The turbulent diffusion of large scale magnetic fields has subsequently been studied in terms of an effective turbulent diffusion coefficient for the large scale field, scaling this coefficient in terms of some power of the magnetic Reynolds number R_M (Blackman & Brandenburg 2002; Brandenburg & Subramanian 2005a; Brandenburg et al. 2008).

Most work on the diffusion of large scale fields has not distinguished between the diffusion of helical vs. non-helical large scale fields. An exception is Yousef et al. (2003), which found that fully helical fields decay more slowly than large scale fields in numerical simulations and discuss this in the context of magnetic helicity evolution. This stimulates a quantitative analytic study to understand just how helical the field must be to be resilient to turbulent diffusion. As we will see, the suppression of helical field diffusion should be interpreted not as an intrinsic suppression of the turbulent diffusion coefficient itself, but as the result of the current helicity correction (Frisch et al. 1975; Pouquet et al. 1976) to the electromotive force which competes with turbulent diffusion.

One important motivation for this study is the potentially dramatic implications for interpreting the origin of large scale magnetic fields in astrophysical rotators (galaxies, disks, stars) as discussed herein. An additional motivation is that jets, particular those in active galactic nuclei (AGN) of parsec scale, exhibit Faraday rotation that is consistent with an ordered large scale helical field (Asada et al. 2008; Gabuzda et al. 2008, 2012). This in turn has led some to conclude that the jets are necessarily magnetically dominated (Lytikov et al. 2005). While Poynting flux dominated models of jets (Li et al. 2001; Lovelace & Romanova 2003; Lynden-Bell 2006; Contopoulos et al. 2006) are plausible, are observed large scale helical fields a definitive signature of a magnetically dominated system?

Although each class of astrophysical source for which the evolution of large scale fields plays a role warrants its own focused study, it is fruitful to investigate simplified problems that potentially identify and elucidate basic principles. In this spirit, we focus on the specific underlying physics of how long it takes for a large scale magnetic field to diffuse in the presence of non-helically forced turbulence when the initial large scale field consists of different helical fractions. We study cases for which the initial field strength does not exceed the kinetic energy as the interiors of astrophysical rotators are typically not magnetically dominated.

In section 2 we derive the basic equations to be solved, drawing from previous work on 21st century dynamo theory and simplifying the set of equations appropriate for the present problem. In section 3 we solve these equations. We discuss the astrophysical implications in section 4 and conclude in section 5.

2 MEAN FIELD DECAY FROM TURBULENT FORCING IN A CLOSED OR PERIODIC SYSTEM

2.1 Derivation of basic equations

Here we derive a system of three differential equations needed to study the decay of large scale magnetic fields of arbitrary helical fraction in a closed or periodic box. These are the equations for the time evolution of (i) large scale magnetic helicity, (ii) small scale magnetic helicity, and (iii) large scale magnetic energy. From these, we will also construct an equation for the evolution of the non-helical large scale field.

To derive the large and small scale magnetic helicity evolution equations we follow standard approaches (Brandenburg & Subramanian 2005a; Blackman 2007) and start with the electric field

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\partial_t\mathbf{A}, \quad (1)$$

where Φ and \mathbf{A} are the scalar and vector potentials. Taking the average (spatial, temporal, or ensemble), and denoting averaged values by the overbar, we have

$$\bar{\mathbf{E}} = -\nabla\bar{\Phi} - \frac{1}{c}\partial_t\bar{\mathbf{A}} \quad (2)$$

Subtracting (2) from (1) gives the equation for the fluctuating electric field

$$\mathbf{e} = -\nabla\phi - \frac{1}{c}\partial_t\mathbf{a}, \quad (3)$$

where ϕ and \mathbf{a} are the fluctuating scalar and vector potentials. Using $\bar{\mathbf{B}} \cdot \partial_t\bar{\mathbf{A}} = \partial_t(\bar{\mathbf{A}} \cdot \bar{\mathbf{B}}) + c\bar{\mathbf{E}} \cdot \bar{\mathbf{B}} - c\nabla \cdot (\bar{\mathbf{A}} \times \bar{\mathbf{E}})$, where the latter two terms result from Maxwell's equation $\partial_t\bar{\mathbf{B}} = -c\nabla \times \bar{\mathbf{E}}$, and the identity $\bar{\mathbf{A}} \cdot \nabla \times \bar{\mathbf{E}} = \bar{\mathbf{E}} \cdot \bar{\mathbf{B}} - \nabla \cdot (\bar{\mathbf{A}} \times \bar{\mathbf{E}})$, we take the dot product of (2) with $\bar{\mathbf{B}}$ to obtain the evolution of the magnetic helicity density associated with the mean fields

$$\partial_t(\bar{\mathbf{A}} \cdot \bar{\mathbf{B}}) = -2c\bar{\mathbf{E}} \cdot \bar{\mathbf{B}} - \nabla \cdot (c\bar{\Phi} \bar{\mathbf{B}} + c\bar{\mathbf{E}} \times \bar{\mathbf{A}}). \quad (4)$$

Similarly, by dotting (3) with \mathbf{b} the evolution of the mean helicity density associated with fluctuating fields is

$$\partial_t\overline{\mathbf{a} \cdot \mathbf{b}} = -2c\overline{\mathbf{e} \cdot \mathbf{b}} - \nabla \cdot (c\overline{\phi \mathbf{b}} + c\overline{\mathbf{e} \times \mathbf{a}}). \quad (5)$$

To eliminate the electric fields from (4) and (5) we use Ohm's law with a resistive term to obtain

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B}/c + \eta \mathbf{J}, \quad (6)$$

where $\mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}$ is the current density and η is the resistivity. Taking the average gives

$$\overline{\mathbf{E}} = -\overline{\mathcal{E}}/c - \overline{\mathbf{V}} \times \overline{\mathbf{B}}/c + \eta \overline{\mathbf{J}}, \quad (7)$$

where $\overline{\mathcal{E}} \equiv \overline{\mathbf{v} \times \mathbf{b}}$ is the turbulent electromotive force. Subtracting (7) from (6) gives

$$\mathbf{e} = (\overline{\mathcal{E}} - \mathbf{v} \times \mathbf{b} - \mathbf{v} \times \overline{\mathbf{B}} - \overline{\mathbf{V}} \times \mathbf{b})/c + \eta \mathbf{j}. \quad (8)$$

Plugging (7) into (4) and (8) into (5) and globally averaging (indicated by brackets) to ignore divergence terms gives, for the small and large scale contributions respectively

$$\frac{1}{2} \partial_t \langle \mathbf{a} \cdot \mathbf{b} \rangle = -\langle \overline{\mathcal{E}} \cdot \overline{\mathbf{B}} \rangle - \nu_M \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle, \quad (9)$$

where $\nu_M = (\eta c^2/4\pi)$. and

$$\frac{1}{2} \partial_t \langle \overline{\mathbf{A}} \cdot \overline{\mathbf{B}} \rangle = \langle \overline{\mathcal{E}} \cdot \overline{\mathbf{B}} \rangle - \nu_M \langle \overline{\mathbf{B}} \cdot \nabla \times \overline{\mathbf{B}} \rangle. \quad (10)$$

To obtain an expression for $\overline{\mathcal{E}}$, we use the 'tau' or 'minimal tau' closure approach for incompressible MHD (Kleeorin et al. 1990; Blackman & Field 2002; Brandenburg & Subramanian 2005a). This means replacing triple correlations by a damping term on the grounds that the EMF $\overline{\mathcal{E}}$ should decay in the absence of $\overline{\mathbf{B}}$. This gives

$$\partial_t \overline{\mathcal{E}} = \langle \partial_t \mathbf{v} \times \mathbf{b} \rangle + \langle \mathbf{v} \times \partial_t \mathbf{b} \rangle = \frac{\alpha}{\tilde{\tau}} \overline{\mathbf{B}} - \frac{\beta}{\tilde{\tau}} \nabla \times \overline{\mathbf{B}} - \overline{\mathcal{E}}/\tilde{\tau}, \quad (11)$$

where $\tilde{\tau}$ is a damping time and

$$\alpha \equiv \frac{\tilde{\tau}}{3} \left(\frac{\langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle}{4\pi\rho} - \langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle \right) \quad \text{and} \quad \beta \equiv \frac{\tilde{\tau}}{3} \langle v^2 \rangle.$$

The time evolution of $\overline{\mathcal{E}}$ can be retained as a separate equation to couple into the theory and solve, but simulations of magnetic field evolution in forced isotropic helical turbulence reveal that a good match to the large scale magnetic field evolution in simulations can be achieved even when the left side of (11) is ignored and $\tilde{\tau} = \frac{1}{v_2 k_f}$, the eddy turnover time associated with the forcing scale (Field & Blackman 2002; Brandenburg & Subramanian 2005b). We adopt that approximation here. Rearranging (11) then gives

$$\overline{\mathcal{E}} = \alpha \overline{\mathbf{B}} - \beta \nabla \times \overline{\mathbf{B}}, \quad (12)$$

Eqs. (4) and (5) then become

$$\frac{1}{2} \partial_t \langle \mathbf{a} \cdot \mathbf{b} \rangle = -\alpha \langle \overline{\mathbf{B}}^2 \rangle + \beta \langle \overline{\mathbf{B}} \cdot \nabla \times \overline{\mathbf{B}} \rangle - \nu_M \langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle \quad (13)$$

and

$$\frac{1}{2} \partial_t \langle \overline{\mathbf{A}} \cdot \overline{\mathbf{B}} \rangle = \alpha \langle \overline{\mathbf{B}}^2 \rangle - \beta \langle \overline{\mathbf{B}} \cdot \nabla \times \overline{\mathbf{B}} \rangle - \nu_M \langle \overline{\mathbf{B}} \cdot \nabla \times \overline{\mathbf{B}} \rangle. \quad (14)$$

Note that the energy associated with the small scale magnetic field does not enter $\overline{\mathcal{E}}$ above. Therefore it does not couple into equations (9) and (10). It appears only as a higher order hyperdiffusion correction (Subramanian 2003) which we neglect because its ratio to the β term in the EMF is $\frac{b^2}{4\pi\rho v^2} \frac{k_1^2}{k_2^2} \ll 1$. However, upon plugging (12) into those equations, the energy associated with the large scale field $\overline{\mathbf{B}}^2$ *does* enter. Therefore we need a separate equation for the energy associated with the energy of the mean field. To obtain this equation we dot $\partial_t \overline{\mathbf{B}} = -c \nabla \times \overline{\mathbf{E}}$ with $\overline{\mathbf{B}}$ and ignore the flux terms to obtain

$$\frac{1}{2} \partial_t \langle \overline{\mathbf{B}}^2 \rangle = -c \langle \overline{\mathbf{B}} \cdot \nabla \times \overline{\mathbf{E}} \rangle = -c \langle \overline{\mathbf{E}} \cdot \nabla \times \overline{\mathbf{B}} \rangle = \langle \overline{\mathcal{E}} \cdot \nabla \times \overline{\mathbf{B}} \rangle - \nu_M \langle (\nabla \times \overline{\mathbf{B}})^2 \rangle = \alpha \langle \overline{\mathbf{B}} \cdot \nabla \times \overline{\mathbf{B}} \rangle - \beta \langle (\nabla \times \overline{\mathbf{B}})^2 \rangle - \nu_M \langle (\nabla \times \overline{\mathbf{B}})^2 \rangle, \quad (15)$$

where the latter two similarities follow from using (7) and (12) and $\overline{\mathbf{V}} = 0$.

Eqs. (13), (14), and (15) form a set that can be solved in a two scale model as long as $\langle v^2 \rangle$ is supplied by steady forcing such that $\partial_t \langle v^2 \rangle \simeq 0$, and $\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle$ remains small. The implications and justification of this latter assumption for present purposes will be discussed in more detail after the results of solving the above equations are presented.

2.2 Two Scale Model and Dimensionless Equations

To extract the essential implications of the coupled Eqs. (13), (14), and (15) for a closed or periodic system, we adopt a standard two-scale model (Blackman & Field 2002; Brandenburg & Subramanian 2005a) and indicate large scale mean quantities with subscript "1" and fluctuating quantities with subscript "2." We write the wave number $k_1 > 0$ to be that

associated with the variation of large scale quantities and that the wave number $k_2 \gg k_1$ to be that associated with the variation of small scale quantities. We also assume that $k_2 = k_f$ where k_f is the forcing wave number at which $v_2^2 = \langle v^2 \rangle$ is maintained to be a constant. We also assume that the turbulence is non-helical (i.e. initially driven with $(\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle) = 0$ and subsequently $|\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle| \ll |\langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle|/(4\pi\rho)$, an assumption to be discussed further in section 3.5. Then Eqs (13), (14), and (15) then become

$$\partial_t H_1 = \left(\frac{2\tilde{\tau}}{3}\right) k_2^2 H_2 \frac{B_1^2}{4\pi\rho} - \frac{2\tilde{\tau}}{3} v_2^2 k_1^2 H_1 - 2\nu_M k_1^2 H_1, \quad (16)$$

$$\partial_t H_2 = -\left(\frac{2\tilde{\tau}}{3}\right) k_2^2 H_2 \frac{B_1^2}{4\pi\rho} + \frac{2\tilde{\tau}}{3} v_2^2 k_1^2 H_1 - 2\nu_M k_2^2 H_2, \quad (17)$$

and

$$\partial_t B_1^2 = \left(\frac{2\tilde{\tau}}{3}\right) \left(\frac{k_2^2 H_2 k_1^2 H_1}{4\pi\rho}\right) - \frac{2\tilde{\tau}}{3} v_2^2 k_1^2 B_1^2 - 2\nu_M k_1^2 B_1^2, \quad (18)$$

where $H_1 = \langle \overline{\mathbf{A}} \cdot \overline{\mathbf{B}} \rangle$ and $H_2 = \langle \overline{\mathbf{a}} \cdot \overline{\mathbf{b}} \rangle$, $B_1^2 = \langle \overline{\mathbf{B}}^2 \rangle$.

We non-dimensionalise these equations by scaling lengths in units of k_2^{-1} , and time in units of $\tau = (k_2 v_2)^{-1} = \tilde{\tau}$, where the latter equality follows since $k_f = k_2$ in our two-scale approach. We define

$$h_1 \equiv \frac{k_2 H_1}{4\pi\rho v_2^2}, \quad h_2 \equiv \frac{k_2 H_2}{4\pi\rho v_2^2}, \quad R_M \equiv \frac{v_2}{\nu_M k_2}, \quad \text{and} \quad M_1 \equiv \frac{\langle \overline{\mathbf{B}}^2 \rangle}{4\pi\rho v_2^2}.$$

Eqs. (16), (17), and (18) can then be respectively written as

$$\partial_\tau h_1 = \frac{2}{3} h_2 M_1 - \frac{2}{3} \left(\frac{k_1}{k_2}\right)^2 h_1 - \frac{2}{R_M} \left(\frac{k_1}{k_2}\right)^2 h_1, \quad (19)$$

$$\partial_\tau h_2 = \frac{-2}{3} h_2 M_1 + \frac{2}{3} \left(\frac{k_1}{k_2}\right)^2 h_1 - \left(\frac{2}{R_M}\right) h_2, \quad (20)$$

and

$$\partial_\tau M_1 = \frac{2}{3} h_1 h_2 \left(\frac{k_1}{k_2}\right)^2 - \frac{2}{3} M_1 \left(\frac{k_1}{k_2}\right)^2 - \left(\frac{2}{R_M}\right) M_1 \left(\frac{k_1}{k_2}\right)^2. \quad (21)$$

Note that for a fully helical large scale field $k_1 |\langle \overline{\mathbf{A}} \cdot \overline{\mathbf{B}} \rangle| = \langle \overline{\mathbf{B}}^2 \rangle$. If we choose a positive large scale helicity $h_1 > 0$, we can drop the absolute value and divide the large scale magnetic energy into a helical fraction $f_1 = (k_1 \langle \overline{\mathbf{A}} \cdot \overline{\mathbf{B}} \rangle) / (\langle \overline{\mathbf{B}}^2 \rangle)$ and a non helical fraction $(1 - f_1)$. Multiplying (19) by k_1/k_2 and subtracting it from (21), the evolution equation for the non-helical magnetic energy of the large scale $M_{1,nh} \equiv M_1 - k_1 h_1/k_2$ becomes

$$\partial_\tau M_{1,nh} = -\frac{2}{3} M_{1,nh} \left(\frac{k_1}{k_2} h_2 + \frac{k_1^2}{k_2^2}\right) - \left(\frac{2}{R_M}\right) M_{1,nh} \left(\frac{k_1}{k_2}\right)^2, \quad (22)$$

which has all decay terms and implies a decay rate even faster than that given by the turbulent diffusivity alone when $h_1, h_2 > 0$. This will be important in understanding the solution plots that follow in the next section.

3 DISCUSSION OF SOLUTIONS

Here we discuss the solutions of Eqs. (19), (20), and (21) for several different cases, assuming that the kinetic energy per unit mass is kept steady, and driven by non-helical forcing at $k_f = k_2$. We identify and derive a minimum helical magnetic energy, in units of kinetic energy, required for slow decay.

3.1 Solutions for fixed initial magnetic energy but varying initial magnetic helicity fraction

Solutions to Eqs. (19), (20), and (21) are shown in Figs. 1a-d for $R_M = 800$ and $k_1 = 1$ and $k_2 = 5$. Each curve in each panel represents a solution with a different initial helical fraction $f_{1,0}$ of large scale magnetic energy, with the initial large scale magnetic energy set to equipartition with the kinetic energy, i.e. $M_{1,0} = 1$. All cases start with $h_2(t=0) = 0$. The six curves of progressively increasing dash spacing correspond to $f_{1,0} = 0.95, 0.7, 0.5, 0.2, 0.04, 0.004$ respectively, so that the solid lined curves correspond to $f_{1,0} = 0.95$ and the widest spaced dashed curves correspond to $f_{1,0} = 0.004$. Fig.1a shows the time evolution of the large scale helical magnetic energy $M_{1,h} = \frac{k_1}{k_2} h_1$ divided by $\frac{k_1 h_c}{k_2}$, where $h_c = k_1/k_2$ is the critical magnetic helicity derived in Sec. 3.3 below. The large scale non-helical magnetic field energy $M_{1,nh}$ (Fig 1b), and the total large scale magnetic energy (Fig 1c) M_1 are normalized to the initial non-helical magnetic energy $M_{1,nh0} = M_{1,0} - \frac{k_1}{k_2} h_1$ and

the initial total magnetic energy $M_{1,0}$ respectively. The evolution of the non-helical magnetic energy follows Eq. (22), derived by subtracting k_1/k_2 times (19) from (21). Fig. 1d shows the time evolution of h_2/h_c .

All curves of Fig 1b show that the non-helical field decays rapidly for all values of $f_{1,0}$. The minimum decay rates occur for $h_2 = 0$. When $h_2 > 0$, the rate of decay is even faster than turbulent diffusion, as can be seen from equation (22) in which the first term on the right side provides enhanced decay for $h_2 > 0$. The rapid decay of the non-helical field in all cases also implies that during the slow decay regimes of M_1 in Fig. 1c, the total field is primarily helical.

The slow decay regimes in Figs 1a and 1c correspond to decay at a microphysical dissipation rate, determined by the last term in Eq. (19). As the plots show, in these regimes $h_1(t)/h_{1,c} > 1$. When instead $h_1(t)/h_{1,c} < 1$, the decay is much more rapid and determined by turbulent diffusion—the penultimate term of (19). Correspondingly, when the helical field is subcritical right from the start, the helical field rapidly decays.

3.2 Solutions for fixed initial magnetic helicity fraction but varying initial magnetic energy and scale ratio

For the solutions to Eqs. (19), (20), and (21) shown in Fig. 2, we used $f_{1,0} = 0.999$ for all curves and varied the initial magnetic energy $M_{1,0}$. We used $k_1 = 1$ and $k_2 = 5$ with $R_M = 800$. From top to bottom the curves in Fig 2a correspond to $\frac{M_{1,0}}{(k_1/k_2)^2} = 20, 5, 1, 0.5, 0.1, 0.01$, respectively. For Fig 2b from top to bottom, the curves correspond to $\frac{M_{1,0}}{(k_1/k_2)^2} = 5, 2, 1, 0.5, 0.1, 0.01$ respectively. The third curve from the top in each panel corresponds to our analytically derived critical value (see next subsection) $M_{1,0} = k_1 h_c / k_2 = (k_1/k_2)^2$. This curve marks the approximate demarcation line between slow and fast decay curves. For curves with $M_{1,0} \gg (k_1/k_2)^2$ decay is slow (resistive), whereas for $M_{1,0} \ll (k_1/k_2)^2$ decay is fast (unfettered turbulent diffusion).

In Fig. 3 we show solutions to Eqs. (19), (20), and (21) for $f_{1,0} = 0.999$ but for different values of k_1/k_2 and R_M than the values used in Fig 2. For Fig. 3a, $k_1 = 1$, $k_f = 10$, $R_M = 8000$, and for Fig. 3b, $k_1 = 1$, $k_f = 20$, $R_M = 8000$, and $f_{1,0} = 0.999$. In each panel, the curves from top to bottom correspond to initial energies $\frac{M_{1,0}}{(k_1/k_2)^2} = 5, 2, 1, 0.5, 0.1, 0.01$ respectively. The third curve from the bottom in each panel again corresponds to our analytically estimated critical curve bounding the slow and fast decay regimes. All curves decay more gradually in Fig. 3b than Fig. 3a. (note the difference in scale of the y – axis in the two figures) because both the turbulent diffusion and the resistive diffusion terms for the case considered in Fig. 3b, are correspondingly reduced by the smaller value of $(k_1/k_2)^2$.

3.3 Derivation of the critical helicity h_c

We now derive the critical helicity h_c and the associated critical helical magnetic energy by noting that the slow decay regime requires the last term on the right of (19) to be at least comparable to the sum of the first two terms on the right. For large R_M , each of those first two terms is separately much larger than the last term. Therefore the first two terms must approximately balance. These same terms also appear in the equation for h_2 , implying that it too evolves slowly (as seen in Fig. 1d.) in the slow decay regime. Since initially we always consider $h_1 > 0$ and $h_2 = 0$, emergence to a slow decay regime implies a rapid evolutionary phase (with negligible dependence on R_M) where the buildup of h_2 leads to an approximate balance between these two terms. But if there is not enough h_1 , then there is not enough supply of magnetic helicity to grow the required h_2 to establish the slow decay regime.

We can estimate the needed amount of h_2 that must be grow by balancing first two terms on the right of either (19) (or (20)) to obtain that

$$h_2 M_1 \simeq (k_1/k_2)^2 h_1. \quad (23)$$

Since $M_1 \simeq k_1 h_1 / k_2$ in the slow decay regime, Eq. (23) gives the result that $h_2 \simeq k_1/k_2$ in this regime. The only possible source of h_2 is h_1 given our initial conditions, therefore the above value of h_2 gives the minimum required of h_1 to achieve the slow decay regime. That is, we must have $h_1 > h_c \equiv k_1/k_2$ for a slow decay regime of the large scale helical field. If ever $h_1 \ll h_c$, the large scale field will decay rapidly. Identification of this critical helicity h_c is the key to explaining the curves shown in Figs. 1, 2, and 3.

The critical magnetic energy associated with h_c is simply $M_{1,c} = k_1 h_c / k_2 = (k_1/k_2)^2$ and this can be substantially below equipartition when $k_1/k_2 \ll 1$. The amount of magnetic energy that decays slowly, $M_{1,slow}$, is then the difference between the magnetic energy contained in the helical field $M_{1,h} = f_1 M_1 = k_1 h_1 / k_2$ and $M_{1,c}$. Dividing this difference by the total magnetic energy then gives the fraction of energy that will decay slowly, namely

$$\frac{M_{1,slow}}{M_1} = \left[\frac{M_{1,h} - M_{1,c}}{M_1}, 0 \right] = \text{Max} \left[f_1 - \frac{k_1^2}{k_2^2} \frac{1}{M_1}, 0 \right]. \quad (24)$$

Eq. (24) shows that most of the initial magnetic energy can decay slowly even if the system is not magnetically dominated or maximally helical. For example, we used $M_{1,0} = 1$ (equipartition between total magnetic and kinetic energy) and $k_1/k_f = 1/5$ for the solution of Fig. 1 so that a fraction $\frac{M_{1,slow}}{M_1} = f_1 - \frac{1}{25}$ of the initial magnetic decays slowly.

3.4 Resilience of helical fields to diffusion is not a reduction in the diffusion coefficient

The resilience of the helical field to turbulent diffusion as shown above, is the result of the current helicity part of the α effect in the language of dynamo theory, not an intrinsic change in the diffusion coefficient β . This is an important distinction because the β term also appears in the non-helical magnetic energy evolution equation where it is not abated by terms involving magnetic helicity, but instead can even be enhanced by them (Eq. 22). The point is that the helical and non-helical large scale fields obey different equations. Parametrization of β in terms of R_M (e.g. Blackman & Brandenburg (2002); Yousef et al. (2003)) can be misleading in this respect, because β itself does not change even when the helical field decays at the resistively limited rate.

3.5 Neglect of kinetic helicity evolution for our choice of initial conditions

In general the α effect is the difference between small scale current and kinetic helicity, but we have not included an equation for the time evolution of the kinetic helicity. If the growth rates of these two helicities were equal so as to keep $\alpha \sim 0$, then the large scale helical field would decay as fast as the non-helical field. We now discuss our justification for ignoring kinetic helicity evolution for our specific choice of initial conditions.

Blackman & Field (2004) and Park & Blackman (2012) showed that the small scale kinetic helicity can grow significantly when the system is initiated with fully helical small scale fields. But the small scale current helicity can only drive small scale kinetic helicity growth and conserve magnetic helicity by inverse cascading, bringing helical magnetic energy up to larger scales. In our present case, any initial magnetic helicity is solely on the large scale to begin with and we now argue that the kinetic helicity associated with the small scale is not expected to grow significantly.

The kinetic helicity growth in the two-scale approximation for a closed system is given by (Blackman & Field 2004)

$$\frac{1}{2} \partial_t H_2^V = \left(\frac{\bar{\mathbf{B}}}{4\pi\rho} \right) \cdot \langle \boldsymbol{\omega} \times \mathbf{b} \rangle (k_2 - k_1) - \nu k_2^2 \langle \mathbf{v} \cdot \boldsymbol{\omega} \rangle + \frac{1}{c\rho} \langle \boldsymbol{\omega} \cdot (\mathbf{j} \times \mathbf{b}) \rangle + \frac{1}{c\rho} \langle \mathbf{v} \cdot \nabla \times (\mathbf{j} \times \mathbf{b}) \rangle, \quad (25)$$

where ν is the viscosity and we assume any non-helical forcing function in the velocity equation does not explicitly contribute. We expect that the first term on the right can only grow kinetic helicity if the contribution from either \mathbf{v} or \mathbf{j} to the correlation comes from their helical part. So we can look at this term in two ways, either focusing on the velocity contribution or the magnetic field contribution. We consider the latter approach first. If we ignore total divergence (surface) terms and spatial derivatives of the cross helicity $\langle \mathbf{v} \cdot \mathbf{b} \rangle$ (given that the latter evolves only via decay $\partial_t \langle \mathbf{v} \cdot \mathbf{b} \rangle = -k_2^2 (\nu + \nu_M) \langle \mathbf{v} \cdot \mathbf{b} \rangle$), then $\langle \boldsymbol{\omega} \times \mathbf{b} \rangle_q = \langle v_s \partial_q b_s \rangle = \langle \mathbf{v} \times \mathbf{j} \rangle_q$ and the first term on the right of (25) can be written $2\bar{\mathbf{B}} \cdot \langle \mathbf{v} \times \mathbf{j} \rangle (k_2 - k_1)$. For a maximally helical small scale field $\mathbf{j} \times \mathbf{b} = 0$ (though $\bar{\mathbf{J}} \times \mathbf{b} \neq 0 \neq \mathbf{j} \times \bar{\mathbf{B}}$) and so the last two terms of (25) vanish. If we consider the case $\mathbf{j} \cdot \mathbf{b} > 0$, then $2\bar{\mathbf{B}} \cdot \langle \mathbf{v} \times \mathbf{b} \rangle k_2 (k_2 - k_1) = 2\bar{\mathcal{E}} \cdot \bar{\mathbf{B}} k_2 (k_2 - k_1)$ and H_2^V could grow positive as quickly as the current helicity $k_2^2 H_2$ since the latter grows at a rate determined by multiplying the first term on the right of Eqn. (9) by k_2^2 .

But since we expect only the helical fraction of the small scale magnetic field to grow kinetic helicity, for non-maximally helical small fields, the factor $\langle \mathbf{v} \times \mathbf{j} \rangle$ would be reduced to $\sim f_2 k_2 \langle \mathbf{v} \times \mathbf{b} \rangle$ where $f_2 \equiv \left(\frac{\langle \mathbf{j} \cdot \mathbf{b} \rangle^2}{\langle \mathbf{j} \rangle^2 \langle \mathbf{b} \rangle^2} \right)^{1/2} \leq 1$ is the helical magnetic field fraction at the small scale. The solutions shown in Fig. 1 indicate that $k_2 H_2 \leq k_1 v^2 / k_2$ even when no drain into H_2^V is considered. Since we would expect the non-helical turbulent forcing to produce $\langle b^2 \rangle \sim \langle v^2 \rangle$, we would then have $f_2 = |k_2 H_2| / \langle b^2 \rangle \leq k_1 / k_2$ for all $f_1 \leq 1$, and the growth term of H_2^V on the right of (25) would be less than k_1 / k_2 times that on the right of (9). In addition, when $\mathbf{j} \times \mathbf{b} \neq 0$, the triple correlation terms on the right of (25) survive. Since these depend only on the non-helical field, we expect them to be decay terms. In addition, an aspect of the kinetic helicity evolution that is not well captured in a two-scale theory is that for typical inertial range spectral power laws, the microphysical viscous diffusion of kinetic helicity diverges with increasing Reynolds number, unlike that of magnetic helicity (e.g. Brandenburg & Subramanian (2005a)). This exacerbates the relative importance of microphysical diffusion in the kinetic helicity equation compared to that in the magnetic helicity equation.

Now consider the second possibility that the first term on the right of (25) survives only when there is a helical velocity field with arbitrary magnetic field. The magnitude of the right of (25) can then be written $2f_v \bar{\mathcal{E}} \cdot \bar{\mathbf{B}} k_2 (k_2 - k_1)$, roughly a factor of the fractional kinetic helicity $f_v \equiv \frac{H_v}{|\boldsymbol{\omega}| |\mathbf{v}|}$ slower than time evolution of the current helicity, $k_2^2 H_2$. (The latter again evolving at a rate determined by multiplying the first term on the right of Eqn. (9) by k_2^2 .) Since the kinetic energy is forced non-helically, $f_v \ll 1$ initially and it is likely that $\bar{\mathcal{E}} \cdot \bar{\mathbf{B}}$ would already saturate before significant H_2^V could grow.

For the above reasons, we therefore expect that H_2^V would not grow significantly to affect our solutions to the specific initial value problem presented in the previous subsection.

4 ASTROPHYSICAL IMPLICATIONS

Our results show that, when subjected to steady non-helical turbulent forcing of kinetic energy at wavenumber k_2 , the non-helical large scale field decays at least as fast as the unfettered turbulent diffusion rate, but the large scale helical field rapidly

decays only when the ratio of its energy to the turbulent kinetic energy drops below the critical value $M_{1,c} = (k_1/k_2)^2$. Above this value, the helical field decays on a microphysical resistive time scale. With the caveat that we have not included boundary terms or buoyancy, this has several provocative implications.

4.1 Presence of observed large scale helical fields does not guarantee a magnetically dominated plasma

Observations of large scale helical fields, such as those detected by Faraday rotation in extragalactic jets (Asada et al. 2008; Gabuzda et al. 2012), or inferred in gamma-ray bursts (Yonetuko 2012), are sometimes interpreted to imply that the field is force-free and therefore dominates the kinetic energy of the system. Our calculations show that this is not necessarily the case: The fact that $M_{1,c} \ll 1$ for $k_1 \ll k_2$, implies that even significantly sub-equipartition helical fields decay on resistive time scales which are typically much longer than dynamical jet time scales. If a jet contained isotropic or quasi-isotropic MHD turbulence, perhaps supplied via an instability at the radial interface between jet and ambient medium (Rossi et al. 2008), then the large scale helical field could survive intact and the system would not necessarily be magnetically dominated. Although the helical large scale field could appear force-free in the sense that $\bar{\mathbf{J}} \times \bar{\mathbf{B}} \simeq 0$, this does not mean that $\mathbf{j} \times \bar{\mathbf{B}}$ or $\bar{\mathbf{J}} \times \mathbf{b}$ vanish. The non-vanishing of the latter are essential in the derivation of (16) and (17), and particularly the appearance of H_2 (the driving due to the current helicity) on the right sides.

We did not include any anisotropic velocity such as shear in our calculations. Nevertheless, our results still demonstrate that the basic point that mere observation of a helical large scale field does not *prove* magnetic energy dominance.

4.2 Rethinking when in situ dynamos are needed to produce large scale fields

Taken at face value, the survival of helical fields to turbulent diffusion may reduce the essentiality of in situ dynamos in systems if boundary terms are unimportant.

Consider the case of galaxies: A long standing criticism of relying on primordial or protogalactic fields as the primary source of galactic fields has been that the mean field would otherwise rapidly diffuse in the galaxy via supernovae induced turbulence if this turbulence were unable to also facilitate competitive exponential growth from a large scale dynamo (Shukurov 2007). Our calculations provide rejuvenated credence to pre-galactic mechanisms of large scale field production (Kulsrud & Zweibel 2008; Subramanian 2010; Widrow et al. 2012), and specifically those that produce sufficiently strong helical fields (Field & Carroll 2000; Copi et al. 2008; Díaz-Gil et al. 2008; Semikoz et al. 2012) because only such helical fields would avoid diffusing over a galactic lifetime in the absence of boundary terms.

Most of the energy in large scale galactic magnetic fields resides in non-helical toroidal fields amplified from poloidal fields by differential rotation. As long as the turbulent decay time for the non-helical field exceeds the linear shear time, then we can expect a predominance of non-helical field in a steady state, even without an in situ dynamo to regenerate the poloidal fields. This is because the helical field provides a minimum value below which the toroidal field cannot drop. The toroidal field enhancement over the poloidal field would be that which can be linearly amplified in a non-helical field diffusion time.

One distinguishing signature of a primordial helical field would be that its magnetic helicity would be of one sign and would thus not show a reversal across the mid-plane of a rotator. In contrast, an in situ large scale dynamo would be expected to produce field whose helicity changes sign across the mid-plane because of the reflection asymmetry of transport coefficients that drive the field growth. This leads to predictions for relative signs of large and small scale helicity and their respective sign reversals for the sun (Blackman & Brandenburg 2003) which seem to be observed (Brandenburg et al. 2011), thus providing evidence for in situ large scale dynamo action in the sun. We lack such measurements of galactic fields, but the absence of a large scale magnetic helicity reversal across the mid-plane would be evidence for primordial galactic fields.

As mentioned, an important caveat is that our calculations do not include buoyancy or other boundary loss terms that could extract large scale helicity at a rate that may still need to be re-supplied from within the rotator. If such terms are important, then both helical and non-helical large scale fields would deplete and an in situ dynamo would be needed for replenishment. But this shifts the focus from turbulent diffusion in conventional wisdom to that of boundary loss terms in assessing the necessity of in situ dynamos. In fact, it may be the boundary loss terms that also facilitate such dynamos in the first place by ejecting small scale helicity that would otherwise quench the large scale dynamo (Blackman & Field 2000; Vishniac & Cho 2001; Shukurov et al. 2006; Sur et al. 2007).

Similar considerations regarding the survival of helical fields would apply for the large scale fields of stars and accretion disks. A long standing debate over whether the large scale fields that power jets must be produced in situ or survive advection in turbulent disks to grow by flux freezing has persisted (Lubow et al. 1994; Lovelace et al. 2009). Our results here would suggest that any helical part of these large scale fields would be more resilient to diffusion and more easily advected. Again boundary loss terms could change the story in that such terms would both justify the need for a dynamo when it comes to ejection of large scale helical fields, while also potentially being essential to its operation by the ejection of small scale magnetic helicity.

Our results would also suggest that, in the absence of boundary terms, fast cycle periods in stars and disks could only

depend on a rapid diffusion of non-helical fields, since the helical fields diffuse too slowly. It should be noted that even in a closed box, when shear is included, cycle periods can arise with $\langle \bar{\mathbf{B}}^2 \rangle$ and $\langle \bar{\mathbf{A}} \cdot \bar{\mathbf{B}} \rangle$ remaining constant (Blackman & Brandenburg 2002). The extent to which such closed volume cycle periods can be fast or remain resistively slow needs more study.

5 CONCLUSION

Using an incompressible two-scale theory that couples magnetic helicity evolution and large scale magnetic energy evolution, we have quantified the relative decay rates of helical and non-helical large scale magnetic fields subject to isotropic non-helical turbulent forcing in a closed (or periodic) system. We identified a critical ratio of the large scale helical magnetic energy to the turbulent kinetic energy given by $M_{1,c} = (k_1/k_2)^2$ above which helical magnetic energy is immune to turbulent diffusion and decays at the microphysical resistive rate. In contrast, we find that the non-helical field always decays at least as fast as the turbulent diffusion rate and approaches a factor of two faster when the small scale helical field reaches its maximum. The calculations herein corroborate the slow decay of helical fields seen in (Yousef et al. 2003) and provide a more complete theoretical explanation by quantifying how much helical field can survive turbulent diffusion and how much decays.

The physical interpretation of our result emerges from basic principles of magnetic helicity dynamics. For a fixed amount of total magnetic helicity, magnetic energy is minimized when magnetic helicity resides on the largest scale. This is the state toward which the system would relax in the absence of kinetic forcing (Taylor 1986). If the system is to conserve magnetic helicity and the large scale helical field is to turbulently diffuse, then the small scale helical field must gain energy beyond that contained in the initial large scale field. The source of the extra energy is the non-helical kinetic forcing. However any growth of small scale magnetic helicity supplies current helicity to the dynamo α coefficient, which in turn regrows large scale helical field—the inverse cascade. A quasi-steady state results when there is enough magnetic helicity in the system such that the inverse cascade from the small scale sends magnetic helicity back to large scales at a rate competitive with of the turbulent diffusion. This state requires an energy in the small scale helical field equal to just k_1/k_2 of the kinetic energy.

Correspondingly, the initial large scale helical field energy required to supply the necessary magnetic helicity is the further reduced fraction $(k_1/k_2)^2$ of the kinetic energy.

All of this leads to a rethinking of the conditions for when in situ dynamos are required in astrophysical objects: If helical fields survive turbulent diffusion, then they would only decay on microphysical resistive time scales, obviating the need for a source of in situ amplification. This may reinvigorate the potential relevance of primordial helical fields for galaxies (Field & Carroll 2000; Copi et al. 2008; Díaz-Gil et al. 2008; Semikoz et al. 2012; Widrow et al. 2012) and advected helical fields for large scale jets in accretion disks. Lovelace et al. (2009). The fact that large scale helical fields can survive even when sub-equipartition with the turbulent kinetic energy also highlights that the mere appearance of a force-free-looking large scale field does not prove that the magnetic energy dominates the kinetic energy.

Our present calculations ignored kinetic helicity evolution, which we argued to be a small contributor to the class of initial value problems studied. We also ignored boundary terms and buoyancy, two key ingredients of real systems. Generalizations that include these ingredients are of great interest for future work. If, in the context of astrophysical disks and stars, buoyancy removes large scale fields without discretion as to their helicity, then in situ dynamos would be needed to sustain large scale fields. Work in 21st century large scale dynamo theory has also been evolving toward the perspective that helicity fluxes, if not boundary terms, may actually be essential for the operation of large scale dynamos by removing the small scale helicity that clogs the evolution of large scale helicity. Thus boundary loss terms for the large scale field may require a dynamo whereas the dynamo itself might require the boundary loss terms for the small scale field.

In this context, note that our initial condition invoked a net helical large scale field without a compensating helical field of opposite sign. This circumstance itself could arise from an MHD dynamo only if the compensating magnetic helicity of opposite sign were dissipated or lost via a boundary flux. Had we started with equal and opposite helical fields on small and large scales subjected to our same non-helical turbulent forcing, the small scale magnetic helicity would inverse cascade and annihilate the large scale magnetic helicity rapidly and the total large scale field would decay rapidly. The slow diffusion of the large scale helical field that we have studied depends on having a finite net magnetic helicity as a starting point.

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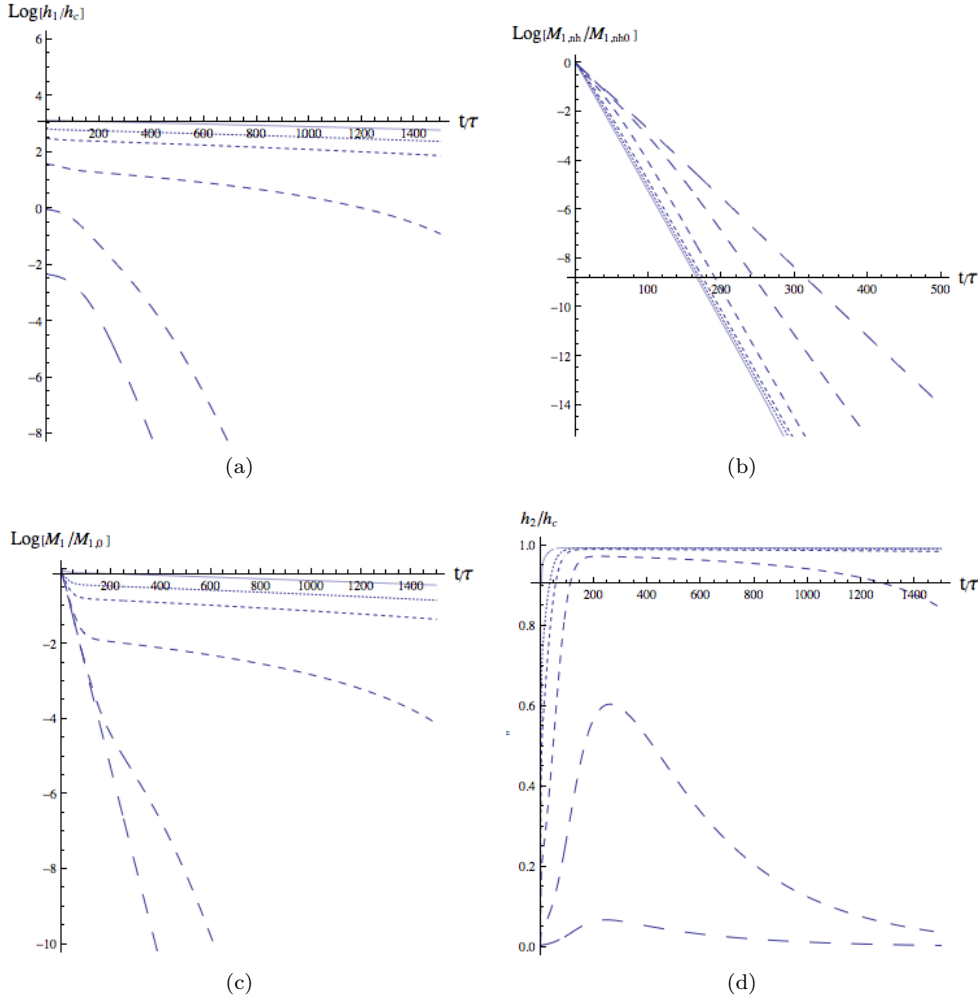


Figure 1. Solutions to Eqs. (19), (20), and (21) for $k_1 = 1$, $k_2 = 5$, $R_M = 800$, constant v_2^2 , and $M_{1,0} = 1$ for all curves. In each panel, the six curves of successively increased dash spacing correspond to $f_{1,0} = 0.95, 0.7, 0.5, 0.2, 0.04, 0.004$ respectively. (a) Dimensionless large scale magnetic helicity, where $h_c = k_1/k_2$ (b) Non-helical magnetic energy in units of the initial non-helical magnetic energy; (c) Total magnetic energy in units of the initial magnetic energy. (d) Dimensionless small scale magnetic helicity h_2 also normalized to h_c .

REFERENCES

- Asada K., Inoue M., Nakamura M., Kamen S., Nagai H., 2008, *ApJ*, 682, 798
- Blackman E. G., 2007, *New Journal of Physics*, 9, 309
- Blackman E. G., Brandenburg A., 2002, *ApJ*, 579, 359
- Blackman E. G., Brandenburg A., 2003, *ApJL*, 584, L99
- Blackman E. G., Field G. B., 2000, *ApJL*, 534, 984
- Blackman E. G., Field G. B., 2002, *Physical Review Letters*, 89, 265007
- Blackman E. G., Field G. B., 2004, *Physics of Plasmas*, 11, 3264
- Brandenburg A., Rädler K.-H., Rheinhardt M., Subramanian K., 2008, *ApJL*, 687, L49
- Brandenburg A., Subramanian K., 2005a, *Phys. Reports*, 417, 1
- Brandenburg A., Subramanian K., 2005b, *A&A*, 439, 835
- Brandenburg A., Subramanian K., Balogh A., Goldstein M. L., 2011, *ApJ*, 734, 9
- Cattaneo F., Vainshtein S. I., 1991, *ApJL*, 376, L21
- Contopoulos I., Kazanas D., Christodoulou D. M., 2006, *ApJ*, 652, 1451
- Copi C. J., Ferrer F., Vachaspati T., Achúcarro A., 2008, *Physical Review Letters*, 101, 171302
- Díaz-Gil A., García-Bellido J., García Pérez M., González-Arroyo A., 2008, *Physical Review Letters*, 100, 241301
- Field G. B., Blackman E. G., 2002, *ApJ*, 572, 685
- Field G. B., Carroll S. M., 2000, *Phys Rev D*, 62, 103008
- Frisch U., Pouquet A., Leorat J., Mazure A., 1975, *Journal of Fluid Mechanics*, 68, 769

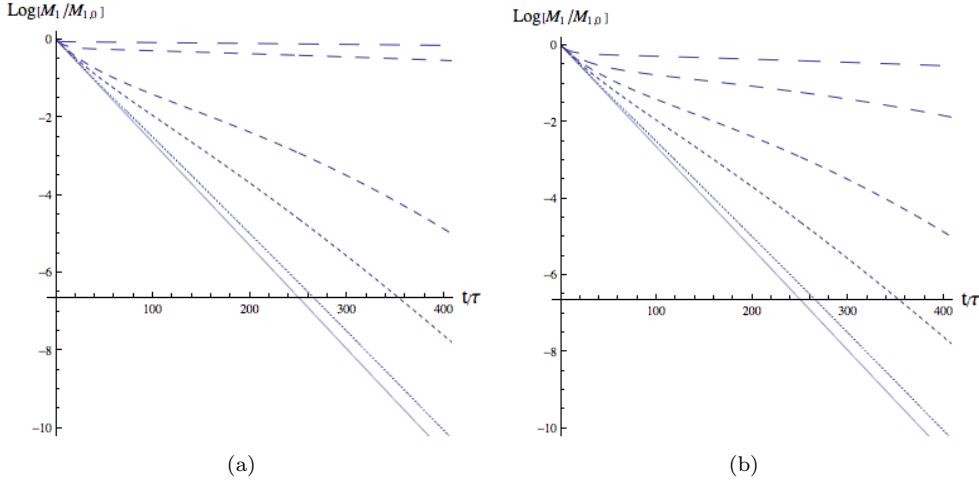


Figure 2. Solutions to Eqs. (19), (20), and (21) for total large scale magnetic energy using a fixed $f_{1,0} = 0.999$, $k_1 = 1$ and $k_2 = 5$ with $R_M = 800$ for various initial magnetic energies $M_{1,0}$. The third curve from the top in each panel corresponds to our analytically derived critical value $M_{1,0} = M_{1,c} = k_1 h_c / k_2 = (k_1 / k_2)^2$. (a) From top to bottom the curves correspond to $\frac{M_{1,0}}{(k_1/k_2)^2} = 20, 5, 1, 0.5, 0.1, 0.01$, respectively. (b) From top to bottom, the curves correspond to $\frac{M_{1,0}}{(k_1/k_2)^2} = 5, 2, 1, 0.5, 0.1, 0.01$ respectively.

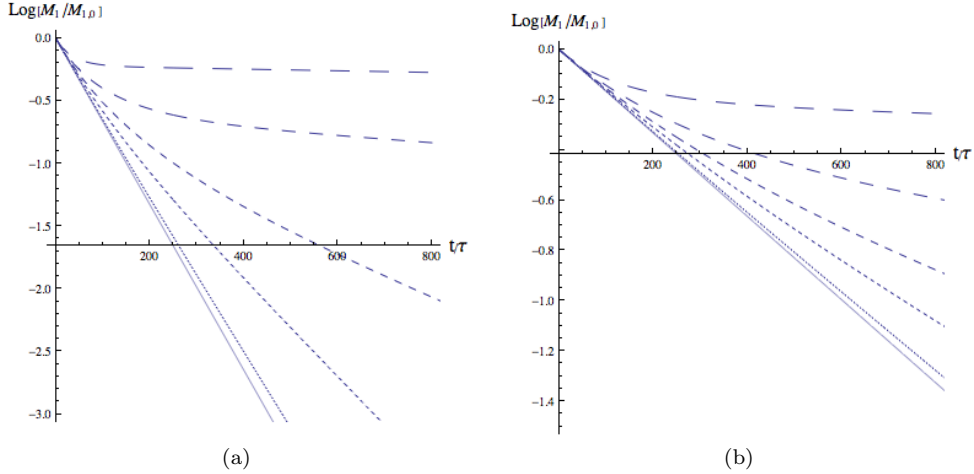


Figure 3. Same as Fig 2b. but for $k_2 = 10$ (left panel), $k_2 = 20$ (right panel), and with $R_M = 8000$ for both panels.

- Gabuzda D. C., Christodoulou D. M., Contopoulos I., Kazanas D., 2012, *Journal of Physics Conference Series*, 355, 012019
 Gabuzda D. C., Vitrichchak V. M., Mahmud M., O’Sullivan S. P., 2008, *MNRAS*, 384, 1003
 Gruzinov A. V., Diamond P. H., 1996, *Physics of Plasmas*, 3, 1853
 Kleeorin N. I., Rogachevskii I. V., Ruzmaikin A. A., 1990, *Sov. Phys. JETP*, 70, 878
 Kulsrud R. M., Zweibel E. G., 2008, *Reports on Progress in Physics*, 71, 046901
 Li H., Lovelace R. V. E., Finn J. M., Colgate S. A., 2001, *ApJ*, 561, 915
 Lovelace R. V. E., Romanova M. M., 2003, *ApJL*, 596, L159
 Lovelace R. V. E., Rothstein D. M., Bisnovaty-Kogan G. S., 2009, *ApJ*, 701, 885
 Lubow S. H., Papaloizou J. C. B., Pringle J. E., 1994, *MNRAS*, 267, 235
 Lynden-Bell D., 2006, *MNRAS*, 369, 1167
 Lyutikov M., Pariev V. I., Gabuzda D. C., 2005, *MNRAS*, 360, 869
 Park K., Blackman E. G., 2012, *MNRAS*, 423, 2120
 Pouquet A., Frisch U., Leorat J., 1976, *Journal of Fluid Mechanics*, 77, 321
 Rossi P., Mignone A., Bodo G., Massaglia S., Ferrari A., 2008, *A&A*, 488, 795
 Semikoz V. B., Sokoloff D. D., Valle J. W. F., 2012, *Journal of Cosmology and Astroparticle Physics*, 6, 8
 Shukurov A., 2007, *Introduction to galactic dynamos*, in *Fluid Mechanics of Astrophysics and Geophysics*, eds. E. Dormy &

- A.M. Soward*, London: Chapman and Hall, p313
Shukurov A., Sokoloff D., Subramanian K., Brandenburg A., 2006, *A&A*, 448, L33
Subramanian K., 2003, *Physical Review Letters*, 90, 245003
Subramanian K., 2010, *Astronomische Nachrichten*, 331, 110
Sur S., Shukurov A., Subramanian K., 2007, *MNRAS*, 377, 874
Taylor J. B., 1986, *Reviews of Modern Physics*, 58, 741
Vishniac E. T., Cho J., 2001, *ApJ*, 550, 752
Widrow L. M., Ryu D., Schleicher D. R. G., Subramanian K., Tsagas C. G., Treumann R. A., 2012, *Space Science Reviews*, 166, 37
Yonetuko D. e. a., 2012, *ApJL*, in press
Yousef T. A., Brandenburg A., Rüdiger G., 2003, *A&A*, 411, 321